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Nomenclature

C_E	Ergun coefficient , $\cong \phi/(180\phi^5)^{1/2}$
C_p	thermal capacity $[L^2T^{-2}Deg]$
d	particle (or pore) size $[L]$
Da	Darcy number $K/(\phi r_3^2)$
J	medium parameter;
	$= 1$: for porous medium
	$= 2$: for solid walls
k_{eff}	porous medium effective thermal conductivity
	$= \phi k_f + (1-\phi)k_s$, $[MLT^{-3}Deg^{-1}]$
k_f	fluid thermal conductivity $[MLT^{-3}Deg^{-1}]$
k_s	solid wall thermal conductivity$[MLT^{-3}Deg^{-1}]$
K	permeability $[L^2]$
K_R	relative thermal conductivity $= k_s/k_{eff}$, $[MLT^{-3}Deg^{-1}]$
l	elementary representative volume length scale $[L]$
L	system length scale (or; r_3) $[L]$
m	geometry parameter ;
	$= 0$: for flat geometry
	$= 1$: for circular geometry
p	pressure $[ML^{-1}T^{-1}]$
P	dimensionless pressure $p/(\rho_f u_D^2)$
Pe	Peclet number $u_D r_3^2/\alpha_{eff}$
Pr_{eff}	effective Prandtl number$(\mu_e)/(\rho_f \alpha_{eff})$
q_{wi}	outer interfacial heat flux $[MLT^{-3}]$
Q_{WI}	absolute dimensionless interfacial heat flux
	$= q_{wi} k_{eff}(T_i - T_w)/r_3 = \lvert \frac{\partial \theta_P}{\partial \eta} \rvert_{int}$
r	radial direction $[L]$
r_1	inner radius of the inner wall $[L]$
r_2	outer radius of the inner wall $[L]$
r_3	inner radius of the outer wall $[L]$
r_4	outer radius of the outer wall $[L]$
$Re_{K^{1/2}}$	Reynolds' number $(u_D \rho_f K^{1/2}/\mu_e)$

t time $[T]$
T temperature $[Deg]$
T_i initial temperature $[Deg]$
T_w wall step temperature $[Deg]$
u volume-averaged axial velocity $[LT^{-1}]$
u_D Darcien velocity (bulk velocity) $[LT^{-1}]$
u_p pore velocity in the axial direction $[LT^{-1}]$
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Creek

α_{eff} porous medium effective thermal diffusivity $= k_{eff}/(C_p\rho)_f$; $[L^2T^{-1}]$
α_i radial grid size scaling factor $(\eta_{i+1}-\eta_i)/(\eta_i-\eta_{i-1})$
α_j axial grid size scaling factor $(\zeta_{j+1}-\zeta_j)/(\zeta_j-\zeta_{j-1})$
α_R relative thermal diffusivity α_s/α_{eff}
α_s solid wall thermal diffusivity $[L^2T^{-1}]$
β wall thickness ratio, η_3/η_4
Δ decriment in any direction
ζ dimensionless axial direction $z\alpha_{eff}/(r_3^2u_D^2)$
η dimensionless radius (r/r_3)
θ dimensionless temperature $(T-T_w)/(T_i-T_w)$
θ_{mix} dimensionless mixing cup temperature
$\int_{\eta_2}^{\eta_3}\theta(\eta)\eta^m U d\eta / \int_{\eta_2}^{\eta^3}\eta^m U d\eta$
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μ viscosity$[ML^{-1}T^{-1}]$
μ_e efficitve viscosity of the fluid $\mu_e=\mu_f{}^{+3.4}_{-2.4}[ML^{-1}T^{-1}]$
ρ density $[ML^{-3}]$
σ thermal capacity ratio $= \frac{\phi(C_p\rho)_f+(1-\phi)(C_p\rho)_s}{(C_p\rho)_f}$
τ dimensionless time $t\alpha_{eff}/r_3^2$
ϕ porosity ratio

Contents

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Jordan University of Science and Technology

May, 1994

TRANSIENT CONJUGATED HEAT TRANSFER IN THE DEVELOPING REGION OF A POROUS CONCENTRIC ANNULUS

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partial fulfillment of the requirements of the

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Chapter 1

Introduction

1.1 Introduction

For the last two centuries, heat transfer through porous media had been a subject of extensive investigations. It covers a wide range of applications, such as Packed-bed chemical reactors, transpiration cooling, geothermal operations, and refrigeration cycles insulation. On the other hand, conjugated heat transfer plays a great rule in determining the thermal behavior of most engineering systems, whenever multimedia systems are involved. Walls of considerable thickness, as well as, those of low relative conductivity, are important examples in this field. Transient behavior of the above applications, becomes important when start/on, and shut/off operations are frequently involved. A very important example of this type of applications, are nuclear reactors.

Although, huge work has been done in this field, it is still lacking to rigorous analytical approaches, as most of the variables are mutually dependent, and can rarely be assumed constant. This encouraged the development of semiheuristic equations, in which micro- and macroscopes are combined to avoid the sweeping generality of the latter, and the complicated details of the former.

In the present work, a numerical solution is carried out to investigate the transient behavior of conjugated heat transfer in a porous media bounded by a concentric annulus. The annulus is assumed to have a uniform temperature initially. Then suddenly it is subject to one of the following boundary conditions:

- Case [O]: Step temperature at the outer side of the outer wall, while the inner side of the inner wall, and the entrance regions are kept adiabatic,
- Case [OE]: Step temperature at the outer side of the outer wall ,and at the entrance region, while the inner side of the inner wall is kept adiabatic,
- Case [I]: Step temperature at the inner side of the outer wall, while the outer side of the outer wall, and the entrance regions are kept adiabatic,
- Case [IE]: Step temperature at the inner side of the inner wall ,and at the entrance region, while the outer side of the outer wall is kept adiabatic

1.2 Literature Review:

Traditional studies in porous media, based primarily its calculations on Darcien models and neglected non-Darcien effects, such as inertia forces and viscous forces along the solid boundaries [1,2,3]. These effects become more significant as the flow velocity increases, or at high porosity media [4,5]. Vafai and Tien [6], utilized the boundary and inertia effects in their study of porous media heat transfer. They showed that those effects were quite significant at high velocities and high velocities, and hence can not be ignored.

Kaviany [7], investigated numerically the laminar flow in a porous channel bounded by an isothermal parallel plates based on the Brinkman model. Vafai and Kim [8], analysed the problem of forced convection in a channel filled with a porous medium and bounded by two parallel plates, and exact solutions were obtained for the velocity and thermal profiles. Poulikakos and Ranken [9], investigated numerically the porosity variation, the inertia forces, and the solid walls effect on both, the momentum, and heat transfer through porous media bounded by parallel plates or circular tube. Al-Nimr et. al. [10] investigated the same problem numerically, in the thermal entrance region of a pipe. Naji [11], investigated also numerically, the velocity and temperature profiles, of the transient problem through porous medium bounded by a concentric annulus.

For the non-porous media case, solutions were obtained analytically and numerically, for different geometries, with and without conjugation. El-Shaarawi and Al-Nimr [12] solved for the fully developed laminar natural convection in open-ended vertical concentric annuli. While, Al-Nimr [13] solved analytically for the transient fully developed free convection in vertical concentric annuli. Yan [14], as well as, Al-Nimr and Hader [15] investigated numerically the transient conjugated heat transfer in non-porous medium pipe flow. The former made use of numerical finite differencing techniques in formulating conjugation at the solid wall-fluid interface, while the later sat a definite formulation at the interface. Schutte el. al. [16] considered the axial conduction in the transient conjugated heat transfere in a pipe with developing laminar flow. The same problem with different boundary conditions, was investigated by Lin and Kuo [17]. Al-Nimr and El-Shaarawi [18], solved analytically for the transient conjugated heat transfer in parallel plate and circular ducts. Hader [19] considered numerically the same problem for a concentric annulus.

To the best of the author's knowledge, transient conjugated heat transfer in porous medium, has not been investigated yet.

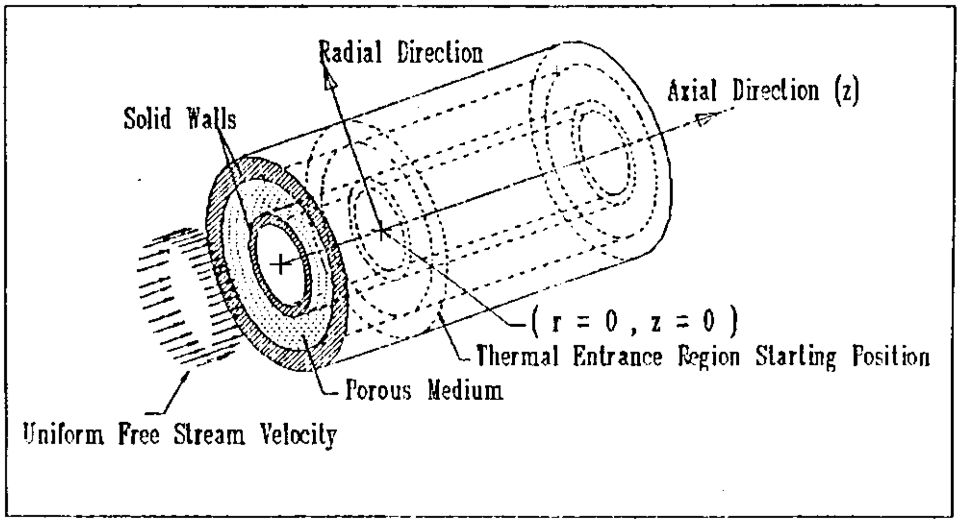

Figure 2.1: Annulus geometry

2.1.2 Porous Medium

The porous medium filling the annulus is assumed to be isotropic. This assumption requires a solid matrix saturated with a fluid yielding any where into a constant properties porous medium in all directions. This includes all characteristic properties of the medium. Obviously, properties can never be infinitesimally constant, therefore, this assumption is valid only for averaged properties over the representative elementary volume. (i.e, if we select any arbitrary volume specimen of the porous media with similar or greater volume then the representative elementary volume, it will have the same averaged properties, no matter where it was picked, or how big it was).

Accordingly, the solid matrix must be of uniform structure in all directions. This requirement reduces the validity of the assumption, as it becomes harder to establish practically. In the actual case, specially near the boundaries channeling phenomenon is more likely to occur, and channeling effect becomes a serious medium for transportation, of mass, momentum, and heat. In some cases, specially

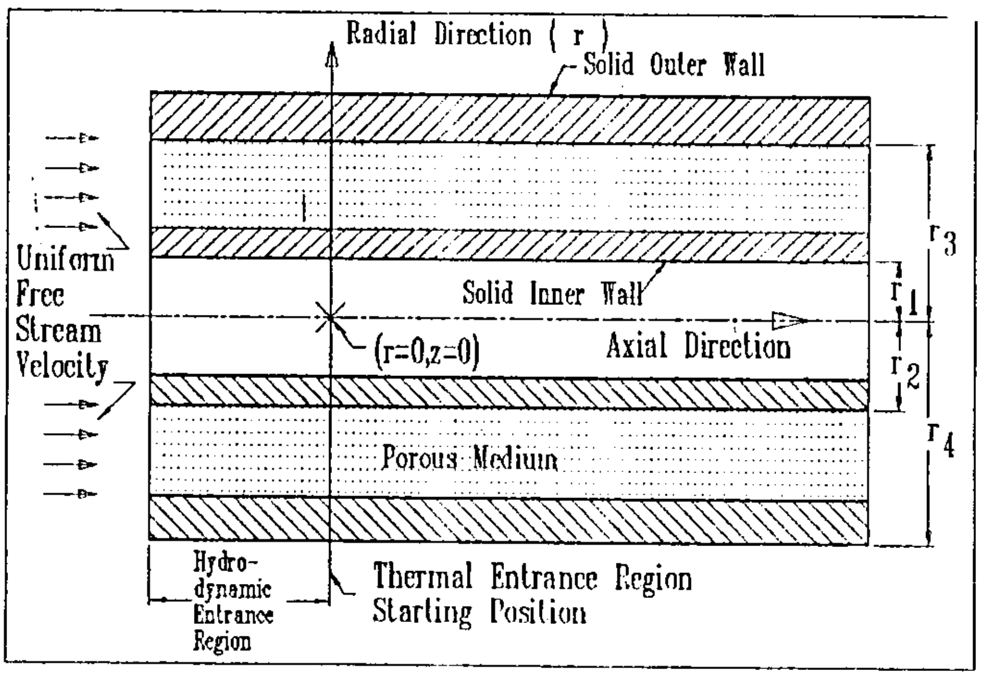

Figure 2.2 : Annulus cross section

for non-consolidated structures, this uncounted transportation, becomes so serious that it causes drastic variations in the thermal behavior of the system considered. The same problem is faced even in consolidated solid structures, as the ratio of inertia to viscous forces becomes lower. However, in the present work, the study will be limitted to consolidated solid structures, with so high inertia to viscous forces ratio, such that, the channeling phenomenon becomes of negligible effect either on mass, momentum, or heat transfer.

The flowing fluid is assumed to be an incompressible newtonian fluid.

2.2 Governing Equations:

2.2.1 Mass Transfer:

Mass is assumed to transfer axisymmetrically in parallel with the axis of the confining annulus. This assumption is valid any where beyond the developing region of the hydrodynamic flow. In porous media the flow becomes hydrodynamically fully developed in the very near entrance of the porous duct [5].

The integral form of the continuity equation for flat geometry can be written as,

$$\int_{r_2}^{r_3} (2r)^m \cdot u \cdot dr = (r_3^{m+1} - r_2^{m+1}) \cdot u_D \qquad (2.1)$$

where;

$m = 0$: for flat geometry, and $m = 1$: for circular geometry.

Note; $u = \phi \cdot u_p$,where u_p is the pore velocity and u is the volume averaged velocity.

2.2.2 Momentum Transfer:

In addition to the fully developed flow assumption, in the preset work inertial flow regime is assumed (i,e; $1 \sim 10 < \frac{\rho_f u_p d}{\mu_e} < 150$) [5]. Such an assumption takes boundary layer effect into consideration, as well as core region at the pore level. It enables us to account for the different effects that caused hydrodynamic behavior effect in determining the hydrodynamic profile, (as the boundary layer becomes of a few particles thickness only [5]). Nevertheless, it still would be of great influence on the thermal profile, as the convective term would grow drastically otherwise. The impermeability assumption is pre-assumed in the impermeable solid walls assumption.

2.2.3 Heat Transfer:

Heat Transfer Through Porous Medium:

Heat transfer in the porous media is thought to be governed by the following macroscopic equation [3]:

$$\sigma \cdot \frac{\partial T_p}{\partial t} + u \cdot \frac{\partial T_p}{\partial z} = \alpha_{eff} \cdot [\frac{1}{r^m} \frac{\partial}{\partial r} r^m \frac{\partial T_p}{\partial r} + \frac{\partial^2 T_p}{\partial z^2}] \tag{2.4}$$

Thermal local equilibrium is assumed, as temperature is introduced in its averaged form. In this assumption the temperature variation within the elementary representative volume is masked, and assumed to be constant. It should be noted here, while adapting this assumption, that the author realizes the existence of the microscopic temperature differences, but the temperature variation over the elementary volume is required to be much much smaller then its variation over the system length scale L .

In addition, it is assumed the above equation, that bouncy effect is negligible, and no heat generation. Also, viscous dissipation is assumed to be negligible, as well as, dispersion effect.

Notice that neglecting the bouncy effect is pre-assumed in the incompressible fluid assumption. And, heat generation is a matter of the actual case of consideration, while, viscous dissipation can only be neglected for fluids of very low viscosity, or for high permeability porous medium. This draws attention, to keep parameters within assumed limits, so that high permeability, and low viscosity does not come in contradiction with the pre-assumed inertial flow regime. As for neglecting the dispersion effect, caution must be made, since dispersion effect depends on most of the problems parameters, such as, Peclet number, porosity, effective conductivity, and effective thermal diffusivity. For low Peclet number, dispersion effect becomes weak [5] at lower porosity levels, hence this assumption may be verified. On the other hand, as the Peclet number decreases, axial conduction neglecting becomes unjustified any more. Therefore, axial conduction must be taken into consideration in the equation. Actually, axial conduction consideration becomes a must in a physical since, if the porous medium is treated as a semi-solid phase.

The boundary conditions for the porous medium will be discussed in the Section 2.2.4.

Heat Transfer Through The Solid Walls:

The governing equation for heat transfer in the solid walls is assumed to be like;

$$\frac{1}{\alpha_s}\frac{\partial T_s}{\partial t} = \frac{1}{r^m}\frac{\partial}{\partial r}r^m\frac{\partial T_s}{\partial r} + \frac{\partial^2 T_s}{\partial z^2} \tag{2.5}$$

In this equation the walls are assumed to be isotropic. Also, axial conduction is taken into consideration. While, no heat generation is assumed.

Notice, that walls in conjugated heat transfer problems play the main rule, in developing the thermal profile of the system considered. The relative high thermal conductivity of the walls, as well as, their considerable thickness, cause axial heat conduction to become a *must*. This supports the previously assumed axial conduction in the porous medium.

Conjugated Heat Transfer :

The unified energy equation, for both solid walls and porous media, would be the union of Equations 2.4and 2.5, and can be written as;

$$\sigma^{2-J}\cdot\frac{\partial T_J}{\partial t} + (2-J)\cdot u\cdot\frac{\partial T_J}{\partial z} = \alpha_J\cdot\left[\frac{1}{r^m}\frac{\partial}{\partial r}r^m\frac{\partial T_J}{\partial r} + \frac{\partial^2 T_J}{\partial z^2}\right] \tag{2.6}$$

where;

$J = 1$, for the porous medium, $J = 2$, for the solid walls, $\alpha_1 = \alpha_{eff}$, $\alpha_2 = \alpha_s$, $T_1 = T_p$ and, $T_2 = T_s$.

At the solid-porous media interface, heat flow is subject to continuity condition;

$$k_{eff} \cdot \frac{\partial T_p}{\partial r} = k_s \cdot \frac{\partial T_s}{\partial r} \tag{2.7}$$

And interfacial temperatures are subject to , no-slip conditions,

$$T_s(r_3) = T_p(r_3)$$

2.2.4 Global Boundary Conditions:

For the annulus shown in Figure 2.1, four different heating mechanisms are investigated. In all cases the whole system is initially at uniform temperature;

At $t = 0.0$, for $r_1 \leq r \leq r_4$, and $0 \leq z < \infty$

$$T(r,z) = T_i$$

Then, for $t > 0$ it becomes subject to one of the following boundary conditions according to the case considered ;

- Case [O]:

  Step temperature at the outer side of the outer wall, while the inner side of the inner wall, and the entrance regions are kept adiabatic. This corresponds to the following boundary and inlet conditions at $t > 0$:

  – at $z = 0$ and, $r_2 < r < r_3$: $T(r,0,t) = T_i$

  – at $z = 0$ and $r_1 < r < r_2$ and $r_3 < r < r_4$: $\frac{\partial T}{\partial z} = 0.0$

  – at $z > 0$ and $r = r_1$: $\frac{\partial T}{\partial r} = 0.0$

  – at $z > 0$ and $r = r_4$: $T(r_4,z,t) = T_w$

- Case [OE]:

  Step temperature at the outer side of the outer wall ,and at the entrance region, while the inner side of the inner wall is kept adiabatic, This corresponds to the following boundary and inlet conditions at $t > 0$:

  – at $z = 0$ and, $r_2 < r < r_3$: $T(r,0,t) = T_w$

- at $z = 0$ and $r_1 < r < r_2$ and $r_3 < r < r_4$: $\frac{\partial T}{\partial z} = 0.0$
- at $z > 0$ and $r = r_1$: $\frac{\partial T}{\partial r} = 0.0$
- at $z > 0$ and $r = r_4$: $T(r_4, z, t) = T_w$

- Case [I]:

  Step temperature at the inner side of the outer wall, while the outer side of the outer wall, and the entrance regions are kept adiabatic. This corresponds to the following boundary and inlet conditions at $t > 0$:

  - at $z = 0$ and, $r_2 < r < r_3$: $T(r, 0, t) = T_i$
  - at $z = 0$ and $r_1 < r < r_2$ and, $r_3 < r < r_4$: $\frac{\partial T}{\partial z} = 0.0$
  - at $z > 0$ and $r = r_1$: $T(r_1, z, t) = T_w$
  - at $z > 0$ and $r = r_4$: $\frac{\partial T}{\partial r} = 0.0$

- Case [IE]:

  Step temperature at the inner side of the inner wall ,and at the entrance region, while the outer side of the outer wall is kept adiabatic, This corresponds to the following boundary and inlet conditions at $t > 0$:

  - at $z = 0$ and, $r_2 < r < r_3$: $T(r, 0, t) = T_w$
  - at $z = 0$ and $r_1 < r < r_2$ and $r_3 < r < r_4$: $\frac{\partial T}{\partial z} = 0.0$
  - at $z > 0$ and $r = r_1$: $T(r_1, z, t) = T_w$
  - at $z > 0$ and $r = r_4$: $\frac{\partial T}{\partial r} = 0.0$

In all the above cases, steady state thermal behavior, is assumed near the end of the axial direction, or;

As $z \to \infty$: $\frac{\partial T}{\partial z} = 0.0$

2.3 Scale analysis:

Selecting the Darcien velocity as a scale variable of axial velocity u;

$$U = \frac{u}{u_D}$$

Choosing the inner radius of the outer wall as a system length scale ;

$$\eta = \frac{r}{r_3}$$

Accordingly , choosing the time scale as the time required for thermal diffusion to penetrate through the system length scale, or in other words, the time required by the system core, to feel thermal changes, on its boundaries;

$$\tau = \frac{t \cdot \alpha_{eff}}{r_3^2}$$

Similarly, choosing the axial length scale, to be as the distance that the slug flow would travel through the penetration time scale;

$$\zeta = \frac{z \cdot \alpha_{eff}}{r_3^2 \cdot u_D}$$

Upon scaling the Darcy law, pressure divergent can be found of order $\frac{\mu_e u_D}{K}$; Therefore, substituting for z scale, $p = \rho_f \cdot u_D^2$, can be chosen as a suitable scale variable of pressure,

$$P = \frac{p}{\rho_f u_D^2}$$

Finally, choosing the dimensionless temperature as, the ratio of temperature deviation from step temperature, to the difference between initial and step rise temperatures;

$$\theta = \frac{T - T_w}{T_i - T_w}$$

2.4 Nondimensional Problem Formulation:

In aid of the proposed scaling variables, the governing equations can be non-dimensionalized to become:

- Non-dimensional Unified Continuity Equation:

$$\int_{\eta_2}^{\eta_3} (2\eta)^m \cdot U \cdot d\eta = (\eta_3^{m+1} - \eta_2^{m+1}) \quad (2.8)$$

- Non-dimensional Unified Momentum Equation :

$$\left(\frac{1}{\eta^m}\frac{\partial}{\partial\eta}\eta^m\frac{\partial U}{\partial\eta}\right) - Da^{-1}\cdot[1 + C_E \cdot Re_{K^{1/2}} \cdot U]\cdot U - \frac{1}{Pr_{eff}}\cdot\frac{dP}{d\varsigma} = 0 \quad (2.9)$$

Subject to

- No-slip condition at the solid boundaries:

$$U(\eta_2) = U(\eta_3) = 0.0$$

- Non-dimensional Unified Energy Equation :

$$\sigma^{2-J}\cdot\frac{\partial\theta_J}{\partial\tau} + (2-J)\cdot u\cdot\frac{\partial\theta_J}{\partial\varsigma} = \frac{\alpha_R^{J-1}}{\eta^m}\cdot\frac{\partial}{\partial\eta}\eta^m\frac{\partial\theta_J}{\partial\eta} + \frac{\alpha_R^{J-1}}{Pe^2}\cdot\frac{\partial^2\theta_J}{\partial\varsigma^2}$$

The general boundary conditions of this equation are:

- at $\tau = 0.0$: $\theta(\eta,\varsigma,0) = 1.0$
- at both $\eta = \eta_2$ and $\eta = \eta_3$: $\frac{\partial\theta_p}{\partial\eta} = K_R\cdot\frac{\partial\theta_s}{\partial\eta}$
  and; $\theta_s = \theta_p$
- as $\varsigma \to \infty$; $\frac{\partial\theta}{\partial\varsigma} = 0.0$

- For $\tau > 0.0$ one of the following cases is imposed:
  - Case [O]:
    - at $\varsigma = 0$ and, $\eta_2 < \eta < \eta_3$: $\theta(\eta, 0, \tau) = 1$
    - at $\varsigma = 0$ and $\eta_1 < \eta < \eta_2$ and $\eta_3 < \eta < \eta_4$: $\frac{\partial\theta}{\partial\varsigma} = 0.0$
    - at $\varsigma > 0$ and $\eta = \eta_1$: $\frac{\partial\theta}{\partial\eta} = 0.0$
    - at $\varsigma > 0$ and $\eta = \eta_4$: $\theta(\eta_4, \varsigma, \tau) = 0$
  - Case [OE]:
    - at $\varsigma = 0$ and, $\eta_2 < \eta < \eta_3$: $\theta(\eta, 0, \tau) = 0$
    - at $\varsigma = 0$ and $\eta_1 < \eta < \eta_2$ and $\eta_3 < \eta < \eta_4$: $\frac{\partial\theta}{\partial\varsigma} = 0.0$
    - at $\varsigma > 0$ and $\eta = \eta_1$: $\frac{\partial\theta}{\partial\eta} = 0.0$
    - at $\varsigma > 0$ and $\eta = \eta_4$: $\theta(\eta_4, \varsigma, \tau) = 0$
  - Case [I]:
    - at $\varsigma = 0$ and, $\eta_2 < \eta < \eta_3$: $\theta(\eta, 0, \tau) = 1$
    - at $\varsigma = 0$ and $\eta_1 < \eta < \eta_2$ and $\eta_3 < \eta < \eta_4$: $\frac{\partial\theta}{\partial\varsigma} = 0.0$
    - at $\varsigma > 0$ and $\eta = \eta_1$: $\theta(\eta_1, \varsigma, \tau) = 0$
    - at $\varsigma > 0$ and $\eta = \eta_4$: $\frac{\partial\theta}{\partial\eta} = 0.0$
  - Case [IE]:
    - at $\varsigma = 0$ and, $\eta_2 < \eta < \eta_3$: $\theta(\eta, 0, \tau) = 0$
    - at $\varsigma = 0$ and $\eta_1 < \eta < \eta_2$ and $\eta_3 < \eta < \eta_4$: $\frac{\partial\theta}{\partial\varsigma} = 0.0$
    - at $\varsigma > 0$ and $\eta = \eta_1$: $\theta(\eta_1, \varsigma, \tau) = 0$
    - at $\varsigma > 0$ and $\eta = \eta_4$: $\frac{\partial\theta}{\partial\eta} = 0.0$

Chapter 3

Numerical Approximation

Introduction

First, the finite difference form of the integral continuity equation , and the momentum equation will presented. Then, the domain shall be discretized. Finally, a logical approach for translating the system of equations into a running computer program will be attempted. Consistency and stability analysis shall be presented in Appendices (A,and B) respectively.

3.1 Solution Methodology

In a closer look at the hydrodynamic, governing equations, it can be seen that the continuity equation in its integral form, is needed as the $n+1^{st}$ equation when solving for n unknown grid point velocities. It will be needed to compensate for the unknown pressure term. Therefore, solving both equations shall be done simultaneously. Also, it can be seen, that the governing equations of the hydrodynamic and thermal profiles are totally . This guides us toward solving for the velocity profile first, using both continuity and momentum equations, and then use the obtained velocity profile, to solve for the thermal profile.

Obviously, The velocity profile is unaffected neither by the axial location, nor by the time location, due to the fully developed flow assumption. Also, it is not influenced by the temperature variation due to the no-dispersion assumption. Therefore, the velocity profile, is only a one step solution. In other words, solving for the velocity profile is needed only once, before it can be used to obtain the thermal profile along any axial location, at any time domain, and for any temperature variation. This suggests that the *simple fully implicit method* is used in obtaining it, making use of the high stability level, and straight forward application, of this method.

In the simple implicit method, the governing equation is centrally finite differenced in one direction, and forwardly differenced in the other marching direction(s). The central differencing produce a equation that can easily solved using the normal *Thomas algorithm*. Then the solution is used next, in the marching direction.

In the present work, there is no marching direction, and only one step solution had to be found. Nevertheless, the continuity equation had to be solved coupled with the momentum equation, and hence the coefficient equation, will not be pure tridiagonal. Therefore, the Thomas algorithm can not be used directly. Another, approach would be to use the *Gaussian elimination*technique in solving the coefficient matrix. But, this on the other hand, would require a huge memory storage, and would reduce the program running time extensively. This would still be acceptable if solution is attempted for a small number of grid points. But, as shown in Section 3.2, the number of grid points is so huge that such an approach becomes rather impossible to carry on with.

In an attempt to solve this problem the coefficient matrix is rearranged as shown in Equation 3.2, so that it became a*down right arrow* matrix, with pressure drop term being the last unknown in it. And, with a little modification on the Thomas algorithm, the auther solved for the pressure drop first, by uppering the coefficient matrix. Then, the obtained pressure drop was used to solve for the velocity components, using direct back substitution. This procedure was repeated to correct for the nonlinear term, until the change in the maximum velocity was less then 1×10^{-10}. Using this modification on the Thomas algorithm, saved us n times the time and memory, used by this technique. The later discussion of discretization, in Section 3.2 would support the modification made by the auther furthermore.

As for the energy equation, it can be seen that it is an elliptic equation in able discretization, could reduce the consistency of the numerical approximation, if it was not employed correctly. The judging criterion for using variable or uniform discretization is the solution predicted behavior. Small grid sizes are needed to capture solution sharp variations, while, large grid sizes are sufficient to reflect smooth solution variations. In other words, grid discretization must come in harmony with the solution variation. The present work gives an rare example, to when variable discretization becomes a *MUST*.

Kaviany, in his discussion of the porous media velocity profiles, [5], noted out " that in the computations of the velocity fields, where the linear dimension of the computational domain is L , grid sizes of $0.1 \cdot K^{1/2}$, are needed for reasonably accurate capturing of the boundary layer phenomenon. If uniform mesh is used then $l/0.1 \cdot K^{1/2}$ nodes are needed. Noting that $L >> l > d >> K^{1/2}$, this requirement becomes rather impossible to meet. An alternative will be variable grids along with a two domain matching similar to matching of inviscid and viscous domains in the boundary-layer analysis. Therefore, a straight forward numerical approach to the Brinkman equation fails to capture the boundary layer."

In agreement with [5], it can be explained why uniform grid computations failed to accept realistic values of Darcy number or Inertia term variables. Actually, porous media flow, specially in the inertia flow regime, is so bulky that it can almost be assumed as a slip flow; And, when applying the no-slip conditions, a drastic change in velocity is expected very close to the boundaries, and almost a uniform flow is present only a few particles thereafter. To come in harmony with such variation, variable discretization becomes a must. Even then, a huge number of nodes is still required.

Therefore, the author has two alternatives, whether to match different domains as suggested by Kaviany [5], or to use a very high stretching factor, to condense as much nodes as possible near the boundaries. In the latter choice, which was adapted in the present work, two problems were faced. First, computational time was proportionally expanded, and storage memory was exhausted. Second, and most important, harmony is lost with the temperature profile, which did not change so drastically near the boundaries. This caused consistency to be seriously damaged.

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